## CSCI 341 Problem Set 1

Puzzle games, solutions, and state diagrams; Automata; Reading words

Due Friday, September 5

## **Games and State Machines**

**Problem 1** (Always an Upper Bound). Let M = (G, L, S, E, A, C) be a directional maze with the set of legal moves  $A = \{ \uparrow, \Rightarrow \}$ . Prove that  $\mathcal{S}(M)$  is finite (there are only finitely many elements) by calculating an upper bound on the number of all possible legal paths through an  $n \times m$  directional maze.

**Problem 2** (Reverse Engineering). Find a Sokoban game that represents state  $s_1$  in abstract state diagram (1). Replace the states in the state diagram with drawings of each state of the Sokoban game.

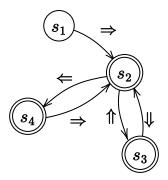


Figure 1: Abstract state diagram (1).

**Problem 3** (Impossibility). Prove that there *does not* exist a directional maze that represents state  $s_1$  in abstract state diagram (1).

## **Reading Words**

**Problem 4** (Repeated Derivatives). Let  $A = (Q, A, \delta, F)$  be an automaton, let  $x \in Q$ , and  $w \in A^*$  and  $a \in A$ . Prove the following identity:

$$\delta(x, wa) = \{z \mid z \in \delta(y, a) \text{ for some } y \in \delta(x, w)\}$$

**Problem 5** (Determinstic Extension). Let  $A = (Q, A, \delta, F)$  be a total deterministic automaton. Let  $x \in Q$  and  $w \in A^*$ . Prove that  $\delta(x, w)$  has exactly one element using Induction on Words.