CSCI 341 Problem Set 8

Introducing Turing Machines

Due Friday, November 7

Don't forget to check the webspace for hints and additional context for each problem!

Problem 1 (ABCs Done Right)**.** Show that the language $\{a^n b^n c^n \mid n \in \mathbb{N}\}$ is decidable.

Solution. □

Problem 2 (Regular Languages are Decidable)**.** Prove that Reg $\subseteq$ Dec.

Solution. □

Problem 3 (With a Counter)**.** Let's consider a variant of the Turing machine that implements an additional *counter*. Its basic machine programs are generated by the grammar

$$E \to \texttt{skip} \mid \texttt{move left} \mid \texttt{move right} \mid \texttt{write}\ \sigma \mid \texttt{count up} \mid \texttt{count dn}$$

and its states have the additional feature of being able to check whether the value of the counter is a particular value. That is, you can write

$$\texttt{if}\ \sigma\ \texttt{and count}\ 5\ \text{(followed by commands)}$$

The output of a Turing machine that operates with a counter is contained in the (single) tape, as usual. Show that every string transformer that can be represented by a Turing machine with a counter is Turing computable in the ordinary sense.

Solution. □

Problem 4 (One Further Step)**.** Show that exponentiation, $f(n, m) \colon \mathbb{N} \times \mathbb{N} \to \mathbb{N}$, is a computable function by reducing it to multiplication.

Solution. □