CSCI 341 Workshop 4

Grammars

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Problem 1 (Same Number)**.** Show that the following language is not regular.

$$L = \{w \in \{0,1\} \mid w \text{ contains the same number of } 0\text{s as it does } 1\text{s}\}$$

Problem 2 (Linear Combination)**.** Show that the following languages are not regular.

(1) $L_1 = \{a^n b^n c^n \mid n \in \mathbb{N}\}$

(2) $L_2 = \{a^n b^{2n-1} \mid n \in \mathbb{N}\}$

(3) $L_3 = \{a^{3n+1} b^{2n-1} \mid n \in \mathbb{N}\}$

Problem 3 (Less Than)**.** Show that the language

$$L_4 = \{a^n b^m \mid n, m \in \mathbb{N} \text{ and } n < m\}$$

is not regular.

Problem 4 (Balanced Parentheses)**.** A string of parentheses, i.e., $)$ and $($, is called *balanced* if every left-parenthese $($ is eventually followed by a unique *matching* right-parenthese $)$. For example, the following strings of parentheses are not balanced:

$$(, \qquad))(), \qquad ((())()$$

but the following strings of parentheses are:

$$\varepsilon, \qquad (), \qquad (())(), \qquad ((())())()$$

Let $A = \{(,)\}$. Prove that the language

$$L = \{w \in A^* \mid w \text{ is balanced}\}$$

is nonregular.

Problem 5 (Palindromes)**.** Now let $A = \{0, 1\}$ and recall that for any word $w = a_1 a_2 \cdots a_n$, we define $w^{\text{op}} = a_n a_{n-1} \cdots a_2 a_1$. Consider the language below:

$$L_{pal} = \{w \in A^* \mid w =^{\text{op}}\}$$

The words in L_{pal} are precisely the *palindromes*. Show that L_{pal} is not regular.